MS Word Export To Multiple PDF Files Software - Please purchase license.How Learning Mathematics Parallels Learning a Second Language and Implications for Teaching

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Abstract

This paper investigates the hypothesis that the language and symbols used to communicate mathematics (ML) can be considered a separate language from natural languages like English. Using definitions and characteristics found in natural languages, the paper investigates when and how these aspects show up in ML. ML is established as a separate language in most features. The paper continues by combining major findings in second language acquisition research with mathematics education research to develop suggestions to improve ML learning and thus improve mathematical learning.

Introduction

Like many subjects, learning mathematics requires learning new vocabulary. However, as teachers of mathematics know, there are more subtle aspects of learning mathematics that have more to do with verbal and written communication than logic or mathematical principles. The question arises, How are language and mathematics related when one is learning or teaching mathematics? Can we consider the language of mathematics to be a second language, separate from English? If yes, then what can we learn from the research about language and mathematics? If no, then what implications does this have on our views and beliefs about mathematics? From this point, this paper will refer to the language of math as ML, while referring to nonmathematical standard language as SL.

ML fits the mold of a language

Definition of a language

First, the claim is that ML acts like a unique language. What is a language? Rotman (1990) gave this as a definition: "A language is a system of relatively *arbitrary symbols* and *grammatical signals* that *change across time* and that members of a community share and use for several purposes: to interact with each other, to *communicate their ideas*, *emotions*, and intentions, and to *transmit their culture from generation to generation* (emphasis added) (Rotman, p. 51)." The claim that mathematical symbols appear to be arbitrary would rarely be debated. Consider the symbol л, the number 6, or the phrase "Cartesian plane". Neither of these mathematical concept has anything to do with the symbols that represent them (л,6, or the letters C,a,r,t,e,s,i,a,n in that order). In fact, in ancient uses of mathematics in Hebrew this is made even more apparent. Numbers did not have unique symbols that represented them. Instead, each number was represented by a letter in the alphabet. For example, the number 1 is א and 2 is ב the first two letters in the Hebrew alphabet (D. Mizraji, personal communication, October 15, 2003; Gal Einai Institute of Israel, 2003).

Rotman (1990) also points out that algebra contains grammatical signals. For example, blank spaces are grammatical in ML. 2xy means 2 times xy where as 2 x y means 2 times y (Rotman, p.51). Also, position is another grammatical structure in ML. As a case in point, exponents and subscripts mean entirely different things. For example, x^2 .is very different from x_2 (Rotman, p. 51). Moreover, the symbols have changed across time. For instance, the printing press was responsible for N! replacing $\underline{|N}$ (Stallings, 2000, p.232). And ^ for exponents has become popular since the typewriter was invented.

However, technology is just one of the reasons for change, most changes in ML simulate changes in other languages. For instance, an author uses a new word or symbol, and if other prominent mathematicians like it enough, then the new word is adopted and eventually standardized. Also, the meaning of zero has changed throughout time. Zero originally was only a place holder, not a number by its own right. Mathematics was quite far along before zero was recognized as a number (Lakoff & Nunez, 2000, p. 366). Thus symbols changed over time and were chosen by the community of scholars (Stallings, p. 232).

ML is used to communicate emotions, like humor. For instance, the famous $\int_0^{cabin} \frac{1}{x} dx$ = a natural log cabin. ML is used to communicate ideas in proofs, explanations, and arguments. The symbols are transmitted by teachers and textbooks from generation to generation. We have now seen that the language of mathematics is a “system of relatively *arbitrary symbols* and *grammatical signals* that *change across time* and that members of a community share and use for several purposes: to interact with each other, to *communicate their ideas*, *emotions*, and intentions, and to *transmit their culture from generation to generation* (Rotman, p. 51). Thus, ML fits Rotman’s definition of a language.

Syntax, Semantic, and Pragmatics

However, languages have more aspects that are not considered in Rotman’s definitions. natural languages can be analyzed with the categories: syntax, semantics and pragmatics. Simplified definitions of these words could be grammar, vocabulary, and language use respectively. Spanos, Rhodes, Dale, and Crandle (1988) argue how ML also has these aspects of language as well.

First, ML has its own syntax. ML has many unique syntactic features such as unique comparatives. A brief sampling of these features is presented here. For example: "Wendy is 'as' old 'as' Jack. Jack is 3 years older than Frank. Frank is 25. How old is Wendy? (Spanos et al., p. 226). " The comparative "as...as" is used for equal relationships. Also, prepositions are used in a unique way. For instance: “What is 4 divided into 9?” or “What is 4 divided by 9?” (Spanos et al., 1988, p. 226). Even the not uncommon question "What is 4 into 9?” collapses the previous phrase and uses a preposition as a verb. As shown in the previous section, Rotman reveals that some spatial positions have grammatical meaning in ML, but they do not have such meaning in SL. Additionally, consider the decimal point in the number 3.14. If you raised the decimal just a little bit ($3 \cdot 14$), a syntactical change, the meaning changes dramatically. Thus we can see that ML has its own syntax.

In semantics, ML has some clear differences with SL. For one thing, many words in ML simply do not exist in SL, such as additive inverse, binomial, denominator. Spanos et al. (1988) discuss semantic phrases unique to ML such as “given that” and “if and only if” (p. 226). These phrases have little or no meaning or use outside of mathematical or logical contexts. They are not used in the everyday language. Moreover, the symbols of math such as /,.*,.-,.+,.<,>,%,# have very different meanings-- if any meaning-- in SL (Spanos et al., p. 227).

Also, some terms in ML overlap with use in SL. Because the terms in ML have completely different meanings in SL, many students get confused (Spanos et al., p.229-31). For example, there are ML terms like rational, imaginary, power, square, inequality that have meanings in SL sometimes quite different from the ML meanings (Spanos et al., p. 226). These ML terms have different and even conflicting meaning in SL. For instance, consider the word imaginary. The word imaginary means nonexistent in SL; whereas imaginary numbers, a ML term, do exist and are used to solve real equations pertaining to real life situations. Imaginary numbers only represent another way to look numbers not a way to look at nonexistent numbers. Students, without any viewpoint of ML as a different language, may see “imaginary” and immediately connect it to “nonexistent” and then dismiss imaginary numbers as unimportant. They may experience problems understanding why they are used or what imaginary numbers really mean. Thus, we see that ML has unique semantics.

Spanos and colleagues (1988) clarify some aspects of pragmatics use in the language of mathematics by looking at the cultural of word problem settings (p. 226-227). Pragmatics is a major aspect of linguistics where language use rather than language structure is considered. In pragmatics, linguists study how a person chooses what to say and how this choice affects the person’s intended audience (Microsoft, 2003).

ML uses language for several reasons different from SL. One aspect of the ML pragmatics is dealing with hypothetical situations. Students must learn to use the language for situations that may never happen; many times they get stuck up wondering "why?" (Spanos et al., p. 232). Furthermore, many word problems assume a certain cultural background, which is innate in the problem (Spanos et al., p. 232).. A further language use of ML is to communicate completely in symbolic form. Thus, we can see that the special language and symbols used in mathematics have these three major components of a language: syntax, semantics, and pragmatics. See Table 1 in Appendix A for more examples of ML fitting the definitions and aspects of a natural language.

ML is distinctive from SL

Raiker (2002) points out that many words have much more precise meanings in mathematical contexts than non-mathematical concepts (p. 45). Studies concerning the words “more” and “less” have shown the separateness of ML from SL. Bernardo (1996) found support for the theory of Kintsch and Greeno (1985) saying that people interpret language differently when they recognize the context is mathematical (p.14). Specifically Bernardo’s study of “more” and “less” comprehension found “people's behaviors that reflect their understanding and interpretation of the information given in the texts vary significantly depending on whether they perceive the task as part of math problem solving or of story reading.” Jones (1982) demonstrated how these words “more” and “less” have meanings that change according to the context. The semantics of the words were found to vary by the mathematical context.

Function over Content Theory

ML differs from SL in its innate reading structure. This author has a theory called the “Function over Content” Theory that may demonstrate other differences between ML and SL. Every word in English is either a function word or a content word. Content words are the verbs, nouns, adjectives, adverbs. The functions words are basically everything else like prepositions and determinates (a, an, the, numbers). Function words play a much more important role in ML than SL.

Function words are the words that speed-readers ignore. Most normal readers pay them little attention. For example, how many of us, when reading our favorite murder mystery, carefully examine of the "the's" or "by's" or "and's" we read in the novel? Perhaps a little experiment will demonstrate this. Don't look back at the last few sentences. Close your eyes and try to remember these sentences. What words do you remember? Take a second to do this before reading on.

Did you remember words like experiment, murder, mystery, read, story? If so, you are in the majority. How many of you remember use of the word "of" twice? In what context was this word used? In SL, “of” does not seem to matter very much; a reader might grasp the overall meaning of the sentences by paying attention only to the content words. Necessity trains us to pay attention to and remember content words.

However, ML is not a language in which content words are important. On the contrary, they are many times irrelevant components of the problem situation. Many times, the content words can be exchanged with anything. Consider the following example: Billy divided 15 apples into 3 groups. How many apples did he have? The words 'Billy' and 'apple' are the words most likely to be remembered and focused on, but they are not the words of importance. They are variables. A math teacher could easily substitute apples for pears and Billy for Bobby.

Mathematics deals with relationships. Relationships *are shown by the function words* such as "into, the, 9, or by". Consider another example: 5 times a number is two more than ten times the number. In these examples, a reader focusing on content words may come away confused. Especially since a vital relationship is shown in the "a" and "the". The relationship indicated is that they are the same number. These function words, which we are used to ignoring in SL, play a vital role in ML. Because function words are the focus in ML and content words the focus in SL, we can once again see a difference between the two languages.

Gerofsky (1996) also gives some examples of differences in ML and SL related to the Function over Content theory. ML seems to be "flouting" the Gricean maxim of quality " ' do not say what you believe to be false' " (Gerofsky, p. 41). Consider the following example.

> A rock dropped from the top of the Leaning Tower of Pisa falls 6 m from the base of the tower. **If** the height of the tower is 59m, at what angle does it lean from the vertical?" The tricky part of the story problem above is the "if"(my emphasis). certainly the Tower of Pisa has been measured. Why use the conditional form here? Is it intended to indicate that the vertical height of the tower is not stable?... Or is it a way of indicating that the referent for the words" the Leaning Tower of Pisa" is not the actual structure in northern Italy, but a hypothetical tower, or stick, or line segment, whose height could be set at any value(say, 59m) and whose slope could be calculated using the given numbers and the Pythagorean Theorem? Again, the writer of the problem seems to be taking pains to say 'Here is a story, ignore this story (Gerofsky, p. 41).

Gerofsky is right that the Tower of Pisa could be anything like a stick or line segment. ML focuses on the relationships that are key to mathematical understanding. Once the relationships are learned, the student can then apply them to real variables in real life.

Good mathematical problem solvers take real life situations and translate them to mathematical relationships, where the relationships matter more than the objects being related. ML is so good at helping students do this translation. By ignoring the content words, or by "here is the story, ignore this story" the student can learn to focus on the function words and the relationships. ML does not follow what H.P. Grice found to be true in SL. ML follows it's own maxims and they differ at times from the Gricean maxims prevalent in SL. This theory could provide more evidence that ML and SL are distinct languages.

Arguments that ML is not a natural language

On the other hand, much of ML is based on symbols, which is in essence a formal language. Consider some of the following views regarding formal vs. natural languages presented by Lakoff and Johnson (1999). "A natural language has phonetics, phonology, and morphology. Formal languages don't (Lakoff & Johnson, p. 259)." "Formal 'languages' have nothing like intonation. (Lakoff & Johnson, p. 259)." "Formal languages are not meaningful. By contrast, natural languages are meaningful, and this meaning arises naturally form everyday human experiences. Moreover, meaning is built into the grammar and lexical structure of natural languages (Lakoff & Johnson, p. 259)." "Whereas the symbols of formal languages must be assigned unique referents, with ambiguity eliminated, expressions in natural languages are normally polysemous; that is, they have multiple meanings that are related by cognitive principles. (Lakoff & Johnson, p. 259)."

Also, consider how we would draw the line between SL and ML? Where does a student begin to communicate in ML? Where does he or she stop communicating in the standard language? Clearly, one could not communicate thought with mathematical language alone (Hersh, 1997). How would one communicate the idea that Picasso's Starry night is inspiring or that one is feeling maudlin? Hersh claims, since mathematics cannot express the same ideas as SL, that ML is only a jargon, "a math lingo" no more.

Rebuttal

To begin with, American Sign Language (ASL) also has absolutely no phonetics yet is a natural language. Similarly, phonology, or the studies of the sound of a language, does not truly apply to ASL. Thus, these aspects of some languages do not have to apply to ML; they do not negate ML's claim to being a language.

Furthermore, the lack of morphology does not apply to ML. One component of morphology is the combining of two separate words into a compound word. This happens in ML in a completely distinctive way from SL. Consider the term 5x and the term 7y. We combine these terms through multiplication and literally think of the new term 35xy as a totally separate, an unlike term from 5x and 7y if you like. Another example arises from topology; a set that is both closed and open is called "clopen."

Also, ML has meaning built into the grammar and lexical structure of it's language. Consider the = sign. In mathematics, this sign grammatically signifies separate distinct phrases to be evaluated and then compared. The = sign creation shows this. Robert Rocorde, an Englishman from the 1500s is the father of the sign, which he selected because it looked like two parallel line segments and nothing can be more equal than two parallel line segments. (Stallings, 2000) Finally, ML has polysemous symbols. Consider x, a symbol that represents both the unknown and multiplication and vector cross product.

Hersh's arguments also are inadequate in rejecting ML as a language. Many times, SL fails us in our attempts to communicate how we feel. For example, often people experience such intense emotions (compassion, fear, love, envy) that words are inadequate for expressing these feelings. Moreover, moving between languages, certain jokes or popular phrases are meaningless out of their original language. Even a good translation fails to carry the true meaning. Do these inadequacies to communicate possible thoughts make English a jargon or dialect only of some other language?

Resolution

To be fair, Lakoff and Johnson were talking about a completely formal language and clearly the language and symbols used in most mathematics does not attempt to be this.

However, we can see that their arguments do not apply for the symbolic portions of normal mathematics. Nevertheless, there could still be some truth in their points about formal languages that do apply to ML. Revisiting the symbol x, we note the polysemous meanings for this symbol are not related to some cognitive principle. The symbol x for multiplication was chosen for religious reason to represent the cross (Stallings, 2000, p. 223).

Also, intonation is another example where ML appears not to be a language. Even ASL has a separate intonation. Signing "shut up" can be done with various degrees of rapidity and forcefulness, which portray quite different connotations. (C. Matthews, personal communication, , April 17th, 2007).

In ML there is no such separate structure as intonation. Of course, one can use intonation while speaking about a mathematical topic but only by borrowing from SL use of intonation. Moreover, the symbolic language of ML has no such structure as intonation. Thus parts of ML are not like a natural language.

In conclusion, at the very least, ML is a subset of SL but a unique one that demonstrates properties of being its own separate language. Despite some differences between ML and some more natural language, Within the content that mathematics attempts to describe, ML is a language just as much as English and Swedish are.

Now, why would the conclusion that mathematics is a separate language matter? What does the claim that ML is distinctive from normal regular English mean? To answer these questions we explore how language, understanding, and mathematical learning are related.

Language, cognition, and mathematical learning

Language and cognition are intertwined

Language and cognition are related. This has been one of the major arguments in psychology in the twentieth century. The book "Language in Thinking" edited by P. Adams contains some articles by some experts in this field. Piaget states, "Thus language and thought are linked in a genetic circle where each necessarily leans on the other in interdependent formation and continuous reciprocal action" (Adams, 1972, p. 179). Piaget, dealing specifically with a concept in ML, also concludes that language is a necessary but not sufficient for the construction of logical operators. Finally, Piaget says that language is a necessary part of learning logical operators (Adams, p. 172-174). Vygotsky says "a word devoid of thought is a dead thing" and "thought is born through words" (Adams, p. 212).

Language affects Cognition

In the same book edited by Adams, a study by Ervin-Trapp showed that language even affects what we think about. Ervin-Trapp found that "as language shifts, content will shift" (Adams,1972, p.257). In the study, the researchers presented the same picture to each subject and asked for words that came to mind. The researchers repeated this experiment but using their subjects' second language. The subjects came up with very different themes. For example, one lady, fluent in both English and Japanese, upon seeing a moon said 'rocket' and 'sky' in English and 'moon viewing' and 'zebra grass' in Japanese (Adams, p.257). We see that the language we are using affects how we think.

Vygotsky presents a study showing how words serve as a directive function to our brain, helping us get to a concept or meaning. Having the word does not guarantee having the concept, but the word can aid in retrieval and direction (Adams, p.215). An implication of this is that language helps us remember concepts. Therefore, a student proficient in using ML may remember the concepts better. Faced with a "real world" problem, he or she will be able to use ML as a directive function to direct him back to the concepts and relationships. The student will then be able to reduce his or her "Leaning Tower of Pisa" (See Gerofsky study above) to a straight line and solve the problem.

Another study by Greenfield, Reich, and Olver looked at the relationship between language and thought. A group of Wolof speakers where compared to French speakers. In Wolof the super-ordinate, or words of "color" and "shape" simply do not exist. They have words for colors and shapes but they don't have the super-ordinate word to help group them. Wolof children were farther behind in the grouping abilities. In other words, the words color and shape helped the French children's cognitive processes (Adams, pp.221-35). Language affects cognition and they are interrelated, what about ML?

Language proficiency and comprehension in mathematics

Many studies and researcher support the idea that ML affects comprehension and performance in mathematics. Raiker (2002) found that incorrect use and understanding of mathematical words leads to flawed understanding in the major concepts of mathematics. Secada (1992) found enough proof to say "language proficiency, no matter how it is measured is related to mathematics achievement (p.639)." Carol A. Bradley (1990) sites several more studies dealing with language and math comprehension issues. Lesh, Landau, and Hamilton (1983) "found that verbalizing strategies and procedures is a common practice during problem solving" (Bradley, p. 16). Gagne and Smith (1962) found that these verbal strategies and procedures produce "fewer errors and increased performance speed and help with cognitive processing" (Bradley, p. 16). Webb, via several studies, found that giving and receiving explanation during word problem solving lead to higher achievement for secondary students (Bradley, p.16). Bradley also reported some studies (Caldwell and Gelding, 1979; Hider, 1982; Knight and Hargis, 1977; Linville, 1976; and Watkin, 1979) showing comprehension of ML in text and word problems is correlated with higher student achievement (Bradley, p. 16). these studies indicate language is related with mathematics proficiency.

Furthermore, Bradley reported a study done by Earp and Tanner (1980) in which they found that students bottom up skills (or understanding of the vocabulary) were adequate but their comprehension of the math terms was poor (Bradley, pp. 16-7). They also stated, " when meaningful contextual clues were provided, students' comprehension of mathematics terms improved (Bradley, p. 17)”. In other words, ML that contains hints, clues, and easier language would be useful. Earp and Tanner suggested that some oral practice with the language would help with the vocabulary (Bradley, p. 16). Knight and Hargis (1977) hypothesized that misunderstanding language aspects such as grammar in ML can lead to poor performance and comprehension (Bradley, pp. 16-7). Taken as a whole these mathematics education literature and research suggest that ML affects the performance and comprehension of mathematical learners.

Bradley (1990) studied whether language comprehension was a good predictor or related to conceptual math achievement. Bradley concluded that students "needed both procedural knowledge and language facility in mathematics for an acceptable level of conceptual achievement." (Bradley, p. 25). Therefore, ML proficiency and understanding of the mathematical concepts together were necessary and sufficient to predict success, but neither by itself.

As these studies have shown, language, and in particular ML, affect mathematical comprehension and performance. Accordingly, what can we learn from language acquisition research that might improve our instruction in mathematics? Also, what can we learn about research specifically regarding language and teaching mathematics?

Implications from research on Language Learning and Math/Language research

Opportunity to Learn/Exposure

When learning a foreign language, exposure in meaningful situations is vital. Krashen’s Input Hypothesis Theory is a seminal language acquisition theory of possible application in the mathematics classroom. "Learners progress along the natural order by understanding input that contains structures a little bit beyond their current level" (Ellis, 1994, p. 273). Consequently, learners can acquire a phrase like" tangent to a curve" when they are exposed to it enough times in speech or contexts to reach understanding. Students can pick up the right vocabulary if they are just exposed to the terms enough times *in understandable situations*. This supports what Earp and Tanner suggest, that ML needs hints, clues, and easier language (Bradley, 1990, p.12).

How can we help the students receive more “comprehensible” input? One way is simply more exposure. Anyone who has attempted to learn a second language strictly by studying the rules without very much exposure to the language in natural settings knows the difficulties of this approach. Communication in the mathematics classroom has been advocated by various researchers and organizations (National Council of Teachers of Mathematics (NCTM) ,1996; NCTM 2000; Krussel, L. 1998; Spanos et al.,1988). Communication is a vital part of truly understanding mathematics.

Additionally, teachers and textbooks can give comprehensible input by providing visuals, examples, real life contexts, etc to make situations easier to understand. Also, the push for alternate forms of assessments such as group assessments, journals, presentations, and the like would allow and encourage the use of mathematical language more (Ziebarth, 2003, p. 177-189). The key is exposure. Students learn what they are given the chance to learn (Hiebert,1999,p.12).

Meaningful use/Contextualize

Possibly the best way to make the mathematics language accessible and meaningful is to put the mathematics in a context. Contextual learning is widely supported by language acquisition research (Brown and Hatch 1995, Pappas and Zecker, 2001; Vann and Fairbairn, 2003). Like communication, this idea of contextualizing the mathematics has also been called for by an increasing number of mathematics education researchers (Witherspoon, 1999, p. 397; see also NCTM 2003b).

Ease the cognitive load

Related to opportunity to learn is the concept of cognitive load. A study by Haynes (1993) shows a few very important elements help make the input comprehensible (Brown, 1995, p. 384). This study dealt with students guessing at nonsense words, to assure that these ESL students had not encountered the words before. Haynes found 1) locally constrained vocabulary clues were better (the clues were found in the same sentence) and 2) the number of other unfamiliar words corresponded to success with an inverse relationship (Brown, p. 384). Mathematics educators may develop text and make curricula choices without realizing the linguistic load that these choices put on their students. This could be especially problematic when students first are exposed to algebraic concepts and symbols. There are a few promising alternatives to previous methods of teaching algebra.

The emerging techniques in the Computer Algebra Systems (CAS) may be a venue that helps ease the cognitive load (see NCTM 2003a; Edwards, 2002; Kramarksi and Hirsch, 2003 for more details about CAS curricula and methodology). Appropriate use of these systems may allow students, who are having linguistic difficulties with the symbolic language, ease into the symbolic language over a period of time without having to remain on the same mathematical conceptual level. Thus allowing the students to see the power of the symbolic manipulations and connect them to meaningful mathematics.

Another idea from algebra and statistics research / curricula emphasizes allowing more time with the informal ideas, progressing into the symbolic reasoning over time (Krussel, 1998, p. 437; MacGregor & Price, 1999, pp. 462-463). For example, Hedden and Langbauer (2003) gradually introduce algebraic concepts into their algebra class. First, they teach through problem solving or a contextual approach. Second, they begin by solving these types of problems through more intuitive methods of charts and graphs.
As they say

> In our former way of teaching, students had to 'get it now or forget it.' They had no context to link with, nor any method other than symbolic reasoning to [verify]… Now

students can transition into more abstract thinking when they are ready. We do not expect mastery right away p.157

Conclusion

In conclusion, the language of mathematics is like a second language. ML has every major aspect of a language. Language affects how we think and in particular language has been shown to be associated with mathematical comprehension. Study of second language acquisition research has found support for many of the reforms proposed by math educators in recent years. These include the following: one, the need to teach mathematics through a problem solving approach with more "real world" situations; two, the need to communicate in mathematics in a variety of ways; three, the use of multiple forms of assessment; and four, the possibilities of Computer Algebra Systems for enhancing the education of students in mathematics.

However, this paper also presents other suggestions that have not received enough attention in mathematics-education research and may help with learning ML. These include the following: one, the need for comprehensible input, possibly meaning a revamping and serious study of textbooks and curriculum for placement of new material and two, finding methods for teaching algebra more naturally so that students ease into the symbolic language through a period of time. Related questions remain open to study. Is the function over content theory valid? Would teaching mathematics using more transparent words lead to greater comprehension in the long run? Also, research needs to continue along the lines many researchers are already pursuing in studying the effectiveness of communication, alternative assessments, and teaching mathematics more contextually.

Appendix A: Other examples for how ML fits aspects of natural languages

Table 1:

Aspect of a natural language	**ML examples**
Semantics	Logarithm, or (inclusive), $\sqrt{\ }$
Syntax	Spacing 2xy compared to $2\times y$, or superscripts implying exponents
Pragmatics	Arguments by contradiction
Phonetics	Really borrowed from SL, but inflection, Say $(5+3)\times 7$ compared $5+(3\times 7)$ with the appropriate pauses
Morphology	Triangle, kilogram
Polysemous Terms	| means divides and part of the absolute value sign, $\frac{a}{b}$ means a fraction (a whole divided into b equal sized parts and you have a of those parts) or a ratio (there are a of something for every b of something else)

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