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In modeling immiscible flow through porous media one ends up with the following class of saturation problems.

$$(0.1) \quad \frac{\partial S}{\partial t} + \nabla \cdot (f(S)\mathbf{u}) - \nabla \cdot (k(S)\nabla S) = Q(S) \text{ on } \Omega \times (0, T_0]$$

with the boundary conditions

$$(0.2) \quad (f(S)\mathbf{u} - k(S)\nabla S) \cdot \mathbf{n} = q \text{ on } \partial\Omega \times (0, T_0]$$

and the initial condition

$$(0.3) \quad S(x, 0) = S^0(x) \text{ on } \Omega$$

with  $\Omega$  a bounded domain of  $\mathbf{R}^n$ ,  $n = 1, 2, 3$ .

$S$  is the saturation of the invading fluid: the ratio of the fluid( water, for instance), in a representative elementary volume, by the volume the voids in the representative elementary volume. The diffusion coefficient  $k = k(S)$  is the conductivity of the medium, which is assumed here to be a function only of  $S$ . The saturation function  $f$  defines the transport term  $\nabla \cdot (f(S)\mathbf{u})$ , where  $\mathbf{u}$  is the Darcy velocity of the fluid(water). We assume that  $\mathbf{u}$  is given and has all the nice properties we need for the analysis. The term  $Q = Q(S)$  represents the source and sink terms.

The main goal of this work is to approximate numerically the solution to problem (0.1)–(0.3). But because equation (0.1) is degenerate, the solution of this problem might be sufficiently regular for numerical applications. We thus replace problem (0.1)–(0.3) by a nondegenerate problem which approximates well the initial problem.

$$(0.4) \quad \frac{\partial S_\beta}{\partial t} + \nabla \cdot (f(S_\beta)\mathbf{u}) - \nabla \cdot (k_\beta(S_\beta)\nabla S_\beta) = Q(S_\beta) \text{ on } \Omega \times (0, T_0]$$

with the boundary conditions

$$(0.5) \quad (f(S_\beta)\mathbf{u} - k(S_\beta)\nabla S_\beta) \cdot \mathbf{n} = q \text{ on } \partial\Omega \times (0, T_0]$$

and the initial condition

$$(0.6) \quad S_\beta(x, 0) = S^0(x) \text{ on } \Omega$$

We show that the solution to the regularized problem converges to the solution to the degenerate problem as the regularization parameter  $\beta$  tends to 0, by establishing error estimates for the perturbation.

It is the solution of the regularized problem that we approximate numerically. We first consider a continuous Galerkin Finite Element Method(when the space

variable is discretized and the time variable remains continuous) applied to the the regularized problem and establish error estimates for this approximation.

Finally we discretize completely the problem, by applying an implicit Euler finite difference for the time variable. If  $\beta$  is the regularization parameter,  $h$  the space discretization parameter, and  $\Delta t$  the time stepping, let  $S$  be the solution to problem (0.1)–(0.3), and  $S_h^n$  the numerical solution at the time step  $n$ , then

$$\sum_0^{N-1} \Delta t \|S(t_{n+1}) - S_h^{n+1}\|_{L^{2+\mu}(\Omega)}^{2+\mu} \leq Ch^{2\gamma} \ln(1/h)^{\frac{\mu}{1+\mu}} \beta^{-\frac{\mu}{1+\mu}} + C \left( \beta \delta^\gamma + \Delta t^{\frac{2+\gamma}{\gamma}} \right)$$

where  $\mu$ ,  $\gamma$ , and  $\delta$  are well defined parameters, and  $\delta$  tends to 0 as  $\beta$  tends to 0.