MTH 111 SUMMER 2 2002

Answer Key to the Practice Problems to Exam 2

1.

(a) $d = \sqrt{((-2)-2)^2 + (8-(-3))^2} = \sqrt{16+121} = \sqrt{137}$

(b) $d = \sqrt{((-\sqrt{2})-0)^2 + (0-\sqrt{3})^2} = \sqrt{2+3} = \sqrt{5}$

2.

(a) $(x-3)^2 + (y+5)^2 = 7^2 = 49$

(b) Length of the diameter: $d = \sqrt{(1+2)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13}$

So the radius is $r = d/2 = \dfrac{\sqrt{13}}{2}$.

The center is the midpoint of the points (- 2, 2) and (1, 4): $\left(\dfrac{-2+1}{2}, \dfrac{2+4}{2}\right) = \left(-\dfrac{1}{2}, 3\right)$.

So the equation of the circle is $\left(x+\dfrac{1}{2}\right)^2 + (y-3)^2 = \dfrac{13}{4}$

3.

Center: (- 3, 0). Radius $r = \sqrt{5}$.

4. (a) $x = 2$; (b) $x = -\dfrac{11}{5}$

5. (a) $x = 3$; (b) $x = 4$.

6.

(a) $(2-6i)(3+i) = (6+6) + (2-18)i = 12 - 16i$

(b) $(7-i) - (2-4i) = 7 - i - 2 + 4i = 5 + 3i$

7.

(a) $\Delta = (-3)^2 - 4(2)(-2) = 9 + 16 = 25 > 0$ We have two real solutions.

$x_1 = \dfrac{-(-3)+\sqrt{25}}{(2)(2)} = \dfrac{3+5}{4} = 2;\quad x_2 = \dfrac{-(-3)-\sqrt{25}}{2\cdot 2} = \dfrac{3-5}{4} = -\dfrac{1}{2}$

(b) $\Delta = (-\sqrt{2})^2 - 4(-2)(1) = 2 + 8 = 10 > 0$. So we have two real solutions given by:

$x_1 = \dfrac{\sqrt{2}+\sqrt{10}}{2};\quad x_2 = \dfrac{\sqrt{2}-\sqrt{10}}{2}$

8.

(a) $x^2 + 4x - 2 = 0 \Leftrightarrow x^2 + 4x = 2 \Leftrightarrow (x+2)^2 - (2)^2 = 2 \Leftrightarrow (x+2)^2 = 6$

Hence $x + 2 = \pm\sqrt{6}$ i.e. $x_1 = -2 + \sqrt{6};\quad x_2 = -2 - \sqrt{6}$

(b) $3x^2 - 4x = 2 \Leftrightarrow x^2 - \dfrac{4}{3}x = \dfrac{2}{3} \Leftrightarrow \left(x - \dfrac{4}{6}\right)^2 - \left(\dfrac{4}{6}\right)^2 = \dfrac{2}{3}$

That is $\left(x - \dfrac{2}{3}\right)^2 = \left(\dfrac{2}{3}\right)^2 + \dfrac{2}{3} = \dfrac{10}{9}$.

Hence $x_1 = \frac{2}{3} + \frac{\sqrt{10}}{3}; \quad x_2 = \frac{2}{3} - \frac{\sqrt{10}}{3}.$

(c) # 25, page 190

$\Delta = (3)^2 - 4(4) = 9 - 16 < 0.$ Yes the equation has two imaginary solutions (complex conjugate)

#26, page 190

$\Delta = (-2)^2 - 4(1)(4) = 4 - 16 = -12 < 0.$ Yes the equation has two imaginary solutions.

9. # 6, page 199

(a) Vertex: $h = \frac{-(-5)}{2} = \frac{5}{2}; \quad k = g\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5 \cdot \frac{5}{2} + 6 = -\frac{25}{4} + 6 = -\frac{1}{4}$

Vertex: $\left(\frac{5}{2}, -\frac{1}{4}\right)$

(b) Line of symmetry: $x = \frac{5}{2}$

(c) Minimum value $-\frac{1}{4}$

10. (a) Vertex : $\left(-\frac{1}{2}, \frac{7}{4}\right)$

(b) Line of symmetry $x = -\frac{1}{2}$

(c) Maximum value $\frac{7}{4} = k$.

11. Easy.

12 Practice with calculator. Examples done in class.

13

2, page 245.

$f(2) = 5.$ So 2 is not a zero.

$f(3) = 29.$ So 3 is not a zero.

$f(-1) = -7.$ Not a zero.

6, page 245

We get $Q(x) = 2x^2 + 3x + 10; \quad R(x) = 29.$

So $P(x) = (2x^2 + 3x + 10)(x - 3) + 29$

#30, page 245

We get $f(2) = 0$. So $f(x) = (x-2)Q(x)$.

Long division of $x^3 + 5x^2 - 2x - 24$ by $x - 2$ gives $Q(x) = x^2 + 7x + 12.$

Thus $x^3 + 5x^2 - 2x - 24 = (x-2)(x^2 + 7x + 12) = (x-2)(x+3)(x+4)$

14.

2, page 253
The zeros are:
3 with multiplicity 2,
- 4 with multiplicity 3, and
0 with multiplicity 4

#4, page 253
We have $f(x) = \left((x-3)(x-2)\right)^2 = (x-3)^2(x-3)^2$
So the zeros are
3 with multiplicity 2, and
2 with multiplicity 2.

16, page 253
Example of polynomial satisfying the conditions
$P(x) = (x+2)(x-3)^2(x+1)$

15.
10, page 268
Vertical Asymptote: $x = 3$
Horizontal asymptote $y = 0$.
y-intercept: (0, 1).
x-interxept: none

20, page 268
Vertical asymptote: $x = -3$
Horizontal asymptote: none
y-intercept : (0, - 3)
x-intercepts: (3,0) and (- 3, 0).

26, page 269
Vertical asymptote: $x = 1$
Horizontal asymptote: None
y-intercept: (0, 4)
x-intercepts: (2, 0) and (- 2, 0).

44, page 269
Example:
$$f(x) = \frac{x+2}{(x+4)(x-5)}$$