MTH 362 FALL 2001

Practice Problems for the Final Exam

Also review Practice problems for Test 1 and for Test 2

A. Homogeneous and non homogeneous second order linear differential equations.

Rule: (i). To find a general solution of a non homogeneous second order differential equation, first find a general solution of the associated homogeneous equation y_h and a particular solution y_p of the non homogeneous equation (any particular solution of that equation). Then: $y = y_h + y_p$.

(b) One (nonzero) solution y_1 is given to us, then we find a second solution in the form $y_2 = u\, y_1$, and get a general solution $y_h = c_1 y_1 + c_2 y_2$.

(c) We have a constant coefficient second order homogeneous equation: we the characteristic equation method to find y_h.

(ii). To find y_p:

(a) if it is given to us, then no problem, we take it.

(b) if it is not given to us and the right hand side is of one of the forms given in table 2.1, page 105, the find y_p in the form given in the second column of the table., with possible modifications as applicable.

I. Modeling Free Oscillations.

1. (a) What is the frequency of vibration of a mass $m = 4\ kg$, on a spring with spring constant(modulus) $20\ nt/m$.

(b) . (a) What is the frequency of vibration of a mass $m = 4\ kg$, on a spring with spring constant(modulus) $45\ nt/m$.

(c) In each case set the corresponding differential equation and solve it.

II Nonhomogeneous second order differential equations.

1. Solve the differential equation

$y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$, given that a particular solution is $y_p = x^3$

2. Solve the initial value problem.

$$\begin{cases} y'' + y = 2x \\ y(0) = -1, \quad y'(0) = 8 \end{cases}$$

given that $y_p = 2x$.

3. Solve the differential equation.

$$y''+y'-6y=-6x^3+3x^2+6x$$

4. Solve the differential equation.

$$y''+8y'+16y=64\cosh 4x$$

5. Solve the initial value problem.

$$\begin{cases} y''+16y=8\cos 3x \\ y(0)=1, \quad y'(0)=0 \end{cases}$$

6. Solve the initial value problem.

$$\begin{cases} y''+4y'=\sin t+\frac{1}{3}s\sin 3t+\frac{1}{5}\sin 5t \\ y(0)=1, \quad y'(0)=\frac{3}{35} \end{cases}$$

B. Matrices and system of linear equations.

1. Let

$$A=\begin{pmatrix} 2 & 1 \\ 1 & 7 \end{pmatrix}, \quad B=\begin{pmatrix} 3 & -4 \\ 2 & 0 \end{pmatrix}, \quad C=\begin{pmatrix} 6 & 1 \\ 0 & -6 \\ 3 & -5 \end{pmatrix}, \quad D=\begin{pmatrix} 4 & -3 \\ -3 & 4 \\ 2 & 10 \end{pmatrix}$$

(a) Find $A-2B$, (b) Find D^t-3C^t, (c) $\left(A^t+4B^t\right)^t$, (d) $B-D$

2. Write the following linear system in matrix form

$$\begin{cases} 5x-3y+z-6t=12 \\ -x+16y+4z-3t=0 \\ 2x-5y+21z+65t=140 \end{cases}$$

3. Let $A=\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$, $C=\begin{pmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & =1 \end{pmatrix}$, $D=\begin{pmatrix} 4 & 3 & 0 \end{pmatrix}$

Calculate the following products, or give reason why they are not defined.
(a) CA, (b) AB, (c) DC, (d) CD, (e) CA^t, (f) C^2, (g) B^2

4. # 11, page 320 in textbook.