3.4 Quadratic and Power Models

Recall that we are able to write an equation for a line if we know two points on the line.

We are able to write an equation for a parabola if we know three points on the parabola.

Problem: Find the function for a parabola if three points on its graph are

(- 2, - 4), (3, 1) and (2, 4).

$y = ax^2 + bx + c$

$(-2,-4)$ $-4 = a(-2)^2 - 2b + c$

$-4 = 4a - 2b + c$

$(3,1)$ $1 = a(3)^2 + 3b + c$

$1 = 9a + 3b + c$

$(2,4)$ $4 = a(2)^2 + 2b + c$

$4 = 4a + 2b + c$

$-4 = 4a - 2b + c \Rightarrow 4 = -4a + 2b - c$

$1 = 9a + 3b + c \Rightarrow 1 = 9a + 3b + c$

$4 = 4a + 2b + c$

$5 = 5a + 5b$

$-1 = -9a - 3b - c$

$4 = 4a + 2b + c$

$3 = -5a - b$

$5 = 5a + 5b$

$8 = 4b$

$2 = b$

$5 = 5a + 5(2)$

$5 = 5a + 10$

$-5 = 5a$

$-1 = a$

Substitute -1 for a and 2 for b in any of the three equations to find that c = 4.

$a = -1$ $b = 2$

$c = 4$

The function for the parabola containing the three given points is $\boxed{y = -x^2 + 2x + 4}$

Oct 13-9:21 AM

If the graph of a set of data has a pattern that ***approximates*** the shape of a parabola or a part of a parabola, a *quadratic function* may be appropriate to model the data.

Also, for equally spaced input values, if ***second differences*** (the differences of *first differences*) are constant, the data may be modeled ***exactly*** by a *quadratic function*.

p 228: 4

The table has inputs, *x*, and outputs for three functions, *f*, *g*, and *h*. Use second differences to determine which function is exactly quadratic, which is approximately quadratic, and which is not quadratic.

X	f(x)	First difference	Second difference
0	0	—	—
2	399	399	—
4	1601	1202	803
6	3600	1999	797
8	6402	2802	803
10	9998	3596	794

f is approximately quadratic.

Oct 13-9:21 AM

X	g(x)	First difference	Second difference
0	2	—	—
2	0.8	-1.2	—
4	1.2	.4	1.6
6	3.2	2	1.6
8	6.8	3.6	1.6
10	12	5.2	1.6

g is exactly quadratic because for equally spaced input values, the second differences are constant.

X	h(x)	First difference	Second difference
0	0	—	—
2	110	110	—
4	300	190	80
6	195	-105	-295
8	230	35	140
10	290	60	25

h is not quadratic.

Remember the **four parts of a model**:

- an equation for the curve that fits the data;
- a statement of what the input variable, usually x, represents along with the measurement units;
- a statement of what the output variable, usually y or f(x), represents along with the measurement units; and
- a statement about the interval of the input values so that a user of the model will know whether he/she is using *interpolation* or *extrapolation*.

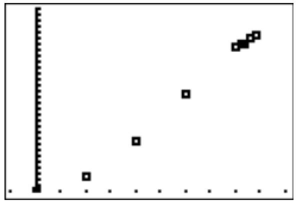
Page 229: 15

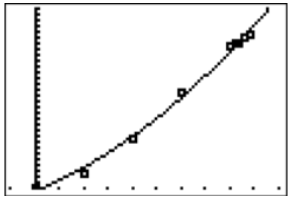
a) x= number of years since 1960

y = median annual income for U.S. males (in dollars)

$y = 6.829\,x^2 + 329.198x + 3550.236$

X is in [0, 44] <u>or</u> data collected for years from 1960 to 2004.





b) In 1997 (x = 37) the model estimates the median annual income of males was $25,080.
We interpolated to find this estimate.

In 2010 (x = 50) the model estimates the median annual income of males was $37,083.
We extrapolated to find this estimate.

c) We are fairly confident about the 1997 figure, but extrapolating will not always give accurate predictions.

Oct 15-9:51 AM

Sometimes a *power function* is an appropriate fit for a set of data points.

If *(log x, log y)* is *linear*, a *power function* is an appropriate model for the data points.

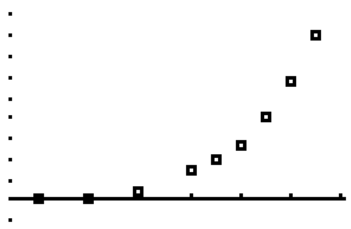
For now, we will follow directions given or try both a quadratic and a power function and visually determine the better fitting function.

There are statistical measurements to measure *goodness of fit* of models to data, but *care must be used in interpreting these measures*.

For example, we can use *r* to measure the goodness of fit for a linear function, but **not** for a quadratic function.

Page 229: 17

a)

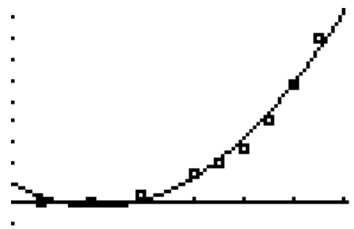


A model with some curvature is appropriate, perhaps a quadratic function.

Oct 13-9:21 AM

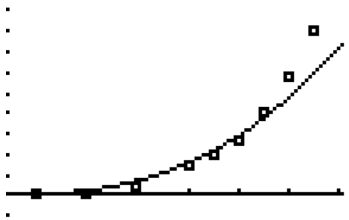
b) For both the power model and the quadratic model,

x = the number of years since 1950
y = national health care expenditures in the US in billions of dollars
$x \in [10, 65]$



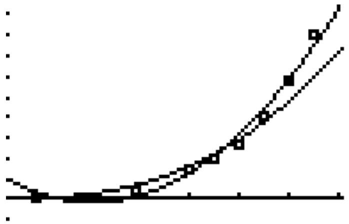
Plot1 Plot2 Plot3
\Y1=2.0253988961
366X^2+-86.72191
670849X+853.8904
49908
\Y2=
\Y3=
\Y4=

quadratic model



Plot1 Plot2 Plot3
\Y1=
\Y2=.03240716023
755X^2.737490901
8414
\Y3=
\Y4=
\Y5=

power model



c) The quadratic model gives an expenditure value for 2010 that is closer to the value given in the table than does the power model.

d) $4707.8 billion

Oct 15-10:02 AM