

LESSON 3 **KEY** – GEOMETRY

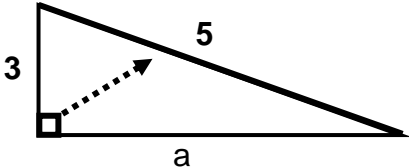
A) THE PYTHAGOREAN THEOREM

The Pythagorean Theorem is used to find the missing side of a right triangle. Remember, the longest side “c” is always across from the right angle.

The Pythagorean Theorem:

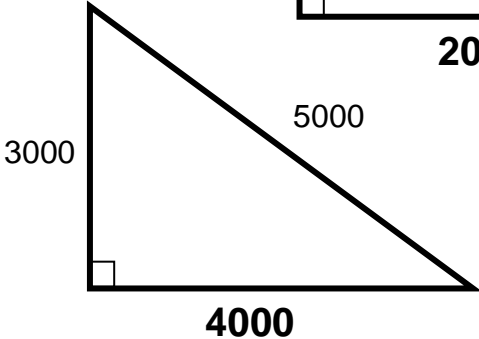
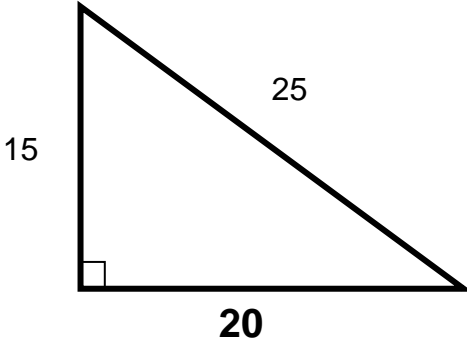
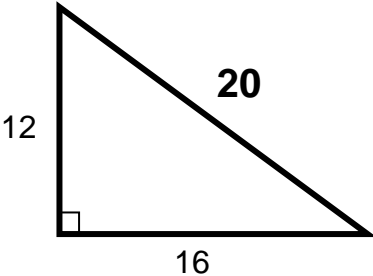
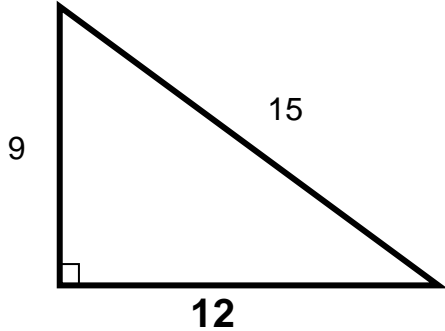
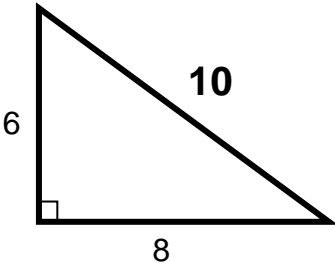
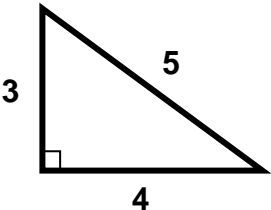
$$a^2 + b^2 = c^2$$

Ex. Find a.



$$\begin{aligned} a^2 + 3^2 &= 5^2 \\ a^2 + 9 &= 25 \\ -9 \quad -9 & \\ a^2 &= 16 \\ a &= \sqrt{16} = 4 \end{aligned}$$

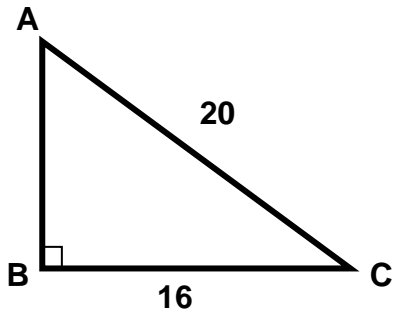
This is a “special” case where you can just use multiples: **3 – 4 – 5**
*If you are lucky enough to get a triangle made up of these **multiples**, then you can do the problem in your head!*



3 – 4 – 5
6 – 8 – 10
9 – 12 – 15
12 – 16 – 20
15 – 20 – 25

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1) Find the length of AB in the right triangle below.



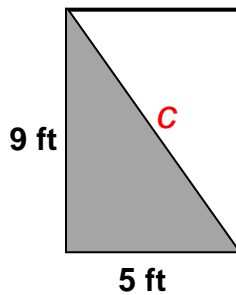
$$\begin{array}{r} 3 - 4 - 5 \\ \times 4 \quad \times 4 \quad \times 4 \\ \hline 12 - 16 - 20 \end{array}$$

12

2) What is the length of a diagonal of a rectangle of length 30 and width 40?

d) 50

$$\begin{array}{r} 3 - 4 - 5 \\ \times 10 \quad \times 10 \quad \times 10 \\ \hline 30 - 40 - 50 \end{array}$$



3) What is the length of a diagonal in the rectangle above?

a) 10.3

Since this is not a multiple of 3-4-5, we use the Pythag. Thm.

$$a^2 + b^2 = c^2$$

$$9^2 + 5^2 = c^2$$

$$81 + 25 = c^2$$

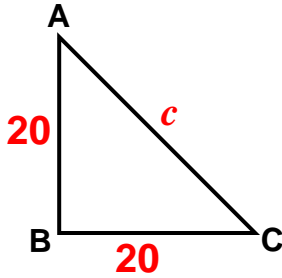
$$106 = c^2$$

$$\sqrt{106} = \sqrt{c^2}$$

$$10.3 = c$$

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4) In right triangle ABC, AB = 20 and BC = 20. Approximately how long is AC?

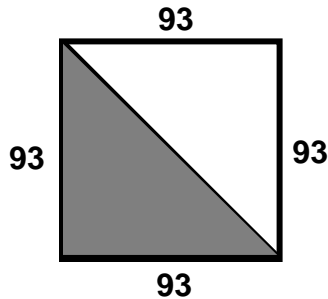


$$\begin{aligned} 20^2 + 20^2 &= c^2 \\ 400 + 400 &= c^2 \\ 800 &= c^2 \\ \sqrt{800} &= c \\ c &= 28.28 \end{aligned}$$

d) 28

Shortcut: If the 2 legs of a right triangle are the same, the hypotenuse will ALWAYS be equal to a leg times $\sqrt{2}$.

In the example above, $AC = 20\sqrt{2} = 28.28$

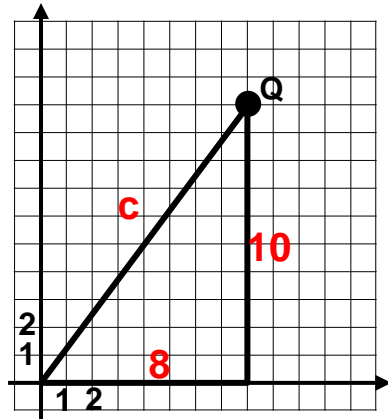


5) What is the length of the diagonal in the square above?

b) $93\sqrt{2}$

Since the 2 legs of the triangles are equal, the hypotenuse is the leg times the square root of 2.

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Draw a right triangle as shown on left.

$$8^2 + 10^2 = c^2$$

$$64 + 100 = c^2$$

$$164 = c^2$$

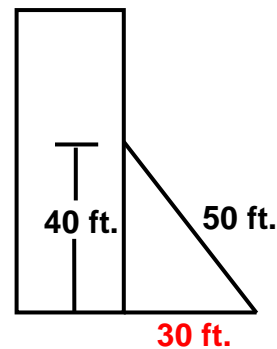
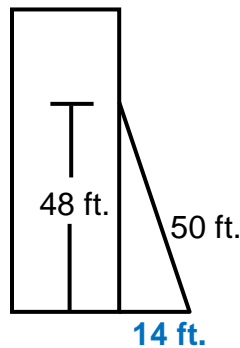
$$c = \sqrt{164}$$

$$c = \sqrt{4} \cdot \sqrt{41}$$

$$c = 2\sqrt{41}$$

6) What is the distance of point Q from the origin?

e) $2\sqrt{41}$



7) A 50 ft. ladder is leaning up against a wall as shown in the drawing above. If the ladder slips down the wall 8 feet, **how much further** will the base of the ladder now be from the wall?

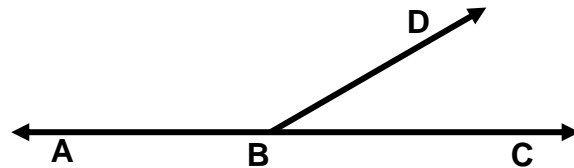
16

If the ladder falls down the wall 8 feet, it will now be at 40 feet. The ladder will still be 50 feet long. So, we have a multiple of 3 – 4 – 5. (3 – 4 – 5) times 10 = 30 – 40 – 50. The ladder will now be 30 feet from the wall. You'll get the same number using the Pythagorean Theorem but it will take longer. However, 30 is not the answer. The question asks how much further will the base of the ladder be from the wall. $30 - 14 = 16$ feet further!

B) ANGLE & TRIANGLE RELATIONSHIPS

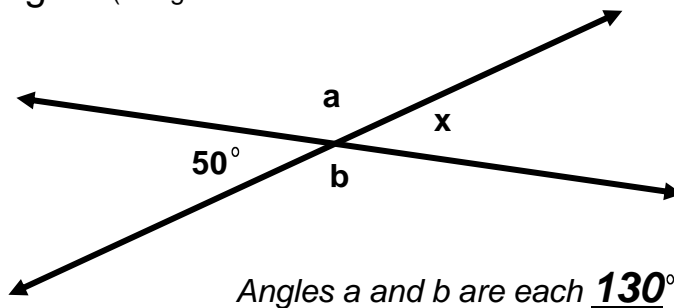
The sum of the angles of a triangle is ALWAYS **180°**.

Angles that form a straight line have a sum of **180°**



If $\angle ABD = 120^\circ$, then $\angle DBC = \mathbf{60^\circ}$.

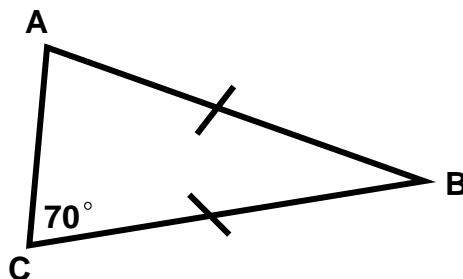
Vertical angles (straight across from each other when two lines intersect) are equal.



$$x = \mathbf{50^\circ}$$

Angles a and b are each **130°**

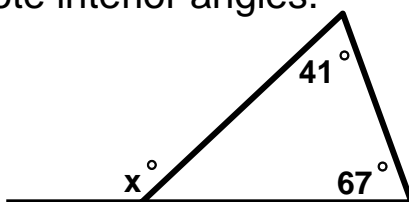
If 2 sides of a triangle are equal, then the **angles** opposite those sides are **also** equal.



$$\text{Angle B} = \mathbf{40}$$

$$\text{Angle A} = \mathbf{70}$$

An **exterior angle** of a triangle is equal to the sum of the two remote interior angles.



$$x = \mathbf{108}$$

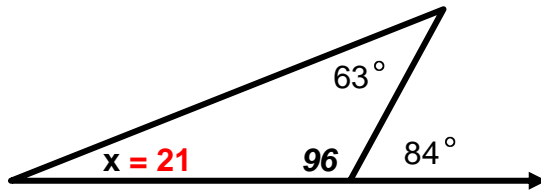
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1) Which of the following could be the measures of angles of a triangle?

c) $20^\circ, 80^\circ, 80^\circ$

These are the only angles whose sum is 180 degrees.

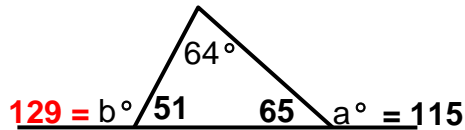
2) Find the measure of x in the picture below:



a) 21°

You don't need to find the 96 in the drawing.

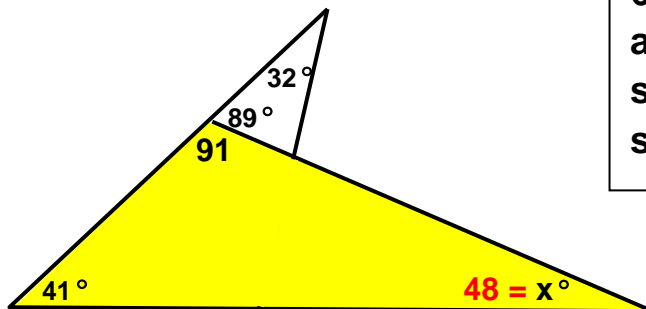
Simply subtract 63 from 84 to get 21.



3) In the diagram above, if $a = 115$ degrees, what is b ?

129

If $a = 115$, then the angle that forms a straight line with it is $180 - 115 = 65$. The 3 angles of a triangle has a sum of 180 so the 3rd angle is $180 - 64 - 65 = 51$. Finally, angle b forms a straight line with 51 so it is $180 - 51 = 129$.

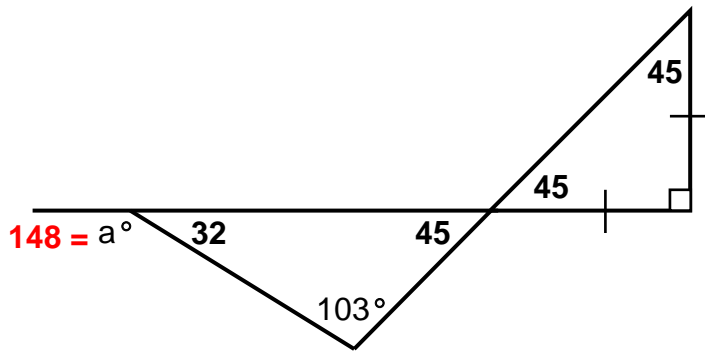


4) What is the value of x?

b) 48

The angle that forms a straight line with 89 is $180 - 89 = 91$. Then the 3 angles of the triangle I have shaded have a sum of 180. So $x = 180 - 91 - 41 = 48$.

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The right triangle has two equal sides so it must have two equal angles. $180 - 90 = 90$. 90 divided by two = 45 for each of the angles.

Since vertical angles are equal, the angle vertical to 45 is also 45.

$180 - 103 - 45 = 32$ for the third angle of the large triangle.

Angle a and the 32 degree angle form a straight line so they must add up to 180. $180 - 32 = 148$.

5) What is the value of a in the picture above?

e) 148

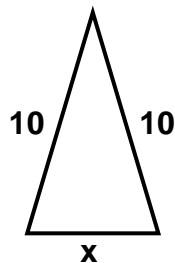
THE TRIANGLE INEQUALITY:

Given the lengths of 2 sides of a triangle, the 3rd side must be between the difference and the sum of the 2 sides.

6) A triangle has sides of lengths 5, 8, and x . Which of the following could be the value of x ? Indicate all such values.

- b) 4.5 c) 6.25 d) 7

The third side must be between the difference and the sum of the other two sides, so x must be between $8 - 5$ and $8 + 5$ or between 3 and 13. The only sides between 3 and 13 are b, c, and d.



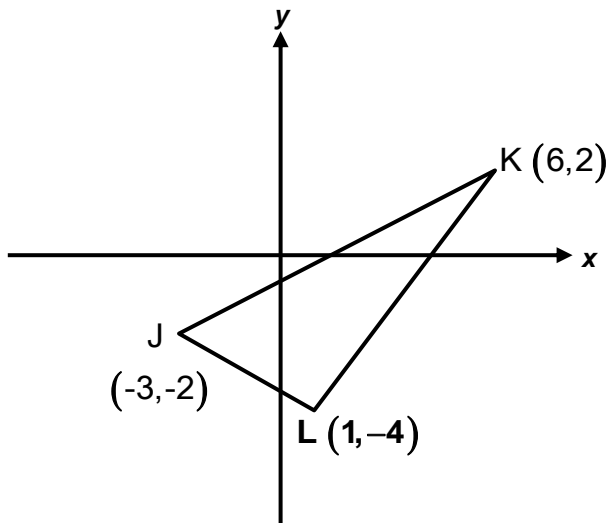
The third side must be between $10 - 10$ and $10 + 10$ or **between 0 and 20**.

7) In the triangle above, x could be any number except:

e) 20

The third side cannot equal 20!

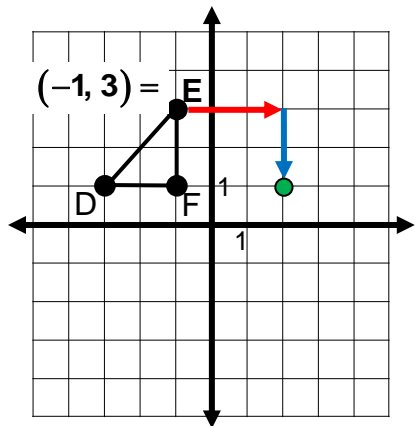
C) TRANSFORMATIONS



A shift of 7 to the right will add 7 to the x-coordinate. A shift of 4 up will add 4 to the y-coordinate.
 $L = (1, -4)$
 $(1 + 7, -4 + 4) = (8, 0)$

- 1) If triangle *JKL* in the *xy*-plane shown above is shifted 7 units to the right and 4 units up, what would be the coordinates of point L after the shift?

e) (8,0)

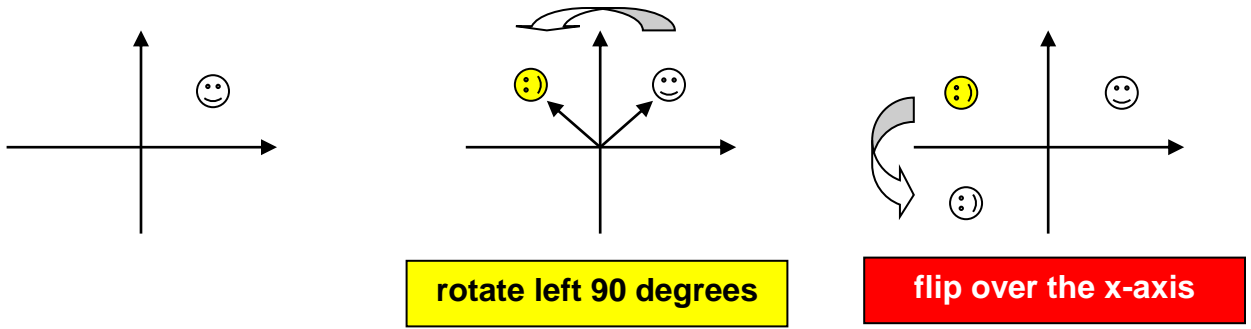


A shift of 3 to the right will add 3 to the x-coordinate. A shift of 2 down will subtract 2 from the y-coordinate.
 $E = (-1, 3)$
 $(-1 + 3, 3 - 2) = (2, 1)$
 Or, just count right 3 and then down 2 and you will land on the point (2,1).

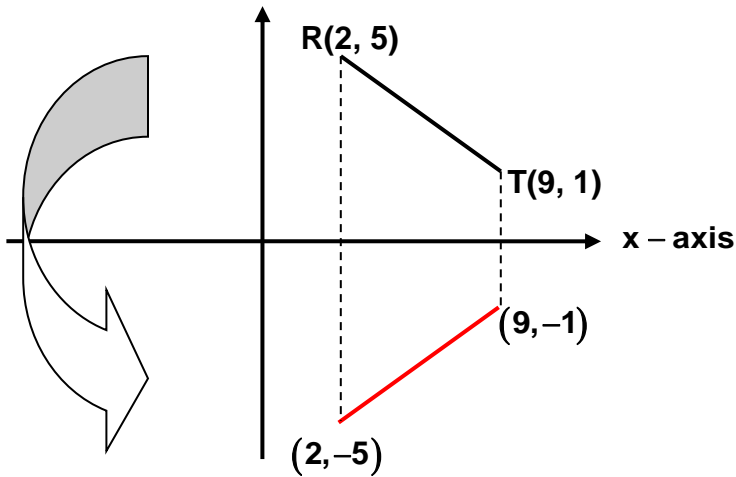
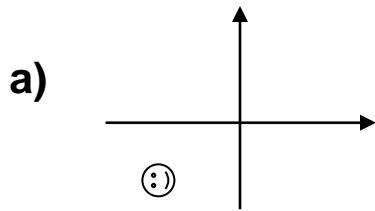
- 2) Triangle *DEF* in the *xy*-plane above will be translated 3 units to the right and then 2 units down. What point will correspond to vertex *E* after these translations?

a) (2,1)

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3) Which of the following graphs shows a 90 degree counter-clockwise rotation of the figure above followed by a reflection over the x-axis?



4) If the segment RT is reflected across the x -axis to the new coordinates $R'T'$, which of the following could be the coordinates of R' and T' ?

c) $(2, -5)(9, -1)$

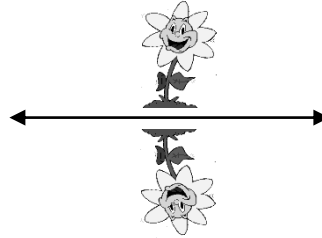
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TYPES OF TRANSFORMATIONS:

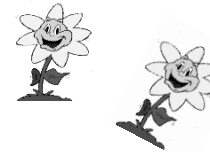
A translation is a **SLIDE**.



A reflection is a **FLIP** over a line.



A rotation is a **TURN** about a point.



A dilation will **INCREASE** the size.



A reduction will **DECREASE** the size.



5) If \overline{AB} is mapped to $\overline{A'B'}$ so that $AB = A'B'$, which of the following is a possible transformation? Choose all that apply.

a) A 180° clock-wise rotation of \overline{AB} about the origin.

d) A translation of \overline{AB} two units to the right and 7 units down.

e) A reflection of \overline{AB} across the x-axis.

$AB = A'B'$ means the size does not change. The only choices that do not change the size are a, d, and e.

D) CIRCLES

A radius is equal to the distance from the center to any point on the circle.

A diameter is the distance across a circle through the center.

A radius = $\frac{1}{2}$ of a diameter. A diameter = 2 x radius,

There are 360 degrees in a circle.

The distance “around” a circle is the Circumference = (diameter) π

Length of an arc: $\frac{\text{number of degrees in the arc}}{360} \cdot d\pi$

Area of a circle = π (radius)²

Area of a sector of a circle: $\frac{\text{number of degrees in the arc}}{360} \cdot \pi r^2$

- 1) A large wheel has a diameter of 30 inches, and a small wheel has a diameter of 20 inches. How many revolutions does the small wheel need to make to travel the same distance that the large wheel travels in 240 revolutions?

d) 360

Hint: 1 revolution = 1 circumference

Large wheel: $C = d\pi = 30(3.14) = 94.2$

240 revolutions of large wheel = $240(94.2) = 22,608$ in.

Small wheel: $C = d\pi = 20(3.14) = 62.8$

Now we need to find out how many revolutions the small wheel must make to reach 22,608.

$62.8x = 22608$

$x = 360$ revolutions

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- 2) If the circumference of a circle is 96 feet, which of the following is closest to the area of the circle?

b) 735

$$C = d\pi$$

$$96 = d(3.14)$$

$$d = 30.6$$

So, the radius = 15.3

$$A = \pi r^2$$

$$A = (3.14)(15.3)^2$$

$$A \approx 735$$

- 3) Harry wants to make a circle graph. He budgets \$150 out of his paycheck of \$450 for food. How many degrees should be in the interior angle for food?

D) 120

$$\frac{150}{450} = \frac{x}{360}$$

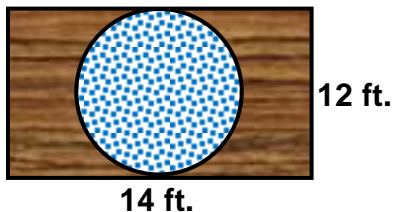
$$450x = 54000$$

$$x = 120$$

Directions: Figure out what fraction of the total is spent on food. Then write a proportion comparing this fraction to **360** degrees in a circle!

- 4) Dawn wants to put a circular rug onto the floor that is a rectangle with dimensions 12 ft. by 14 ft. What is the area of the largest rug that will fit into the room?

b) 36π



In order for the circular rug to fit, it can only have a diameter of the shorter dimension = 12 feet, so the radius = 6 ft.

$$A = \pi r^2 = \pi 6^2 = \pi 36 \text{ or } 36\pi$$

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- 5) In a circle graph a sector of 100 degrees represents an investment of 25 million dollars. What is the value of the total investment?

e) 90 million

Notice the difference between question #5 and #6. In #5, we do not know the total investment. In #6 we do know the total investment.

$$\frac{\text{degrees}}{\text{money}} = \frac{\text{degrees}}{\text{money}}$$

$$\frac{100}{25} = \frac{360 \text{ (total)}}{x}$$

$$100x = 9000$$

$$x = 90$$

- 6) In a circle graph a sector of 100 degrees represents the amount a company spent on employee salaries. If the entire budget of the company is 25 million, how much was spent on employee salaries?

a) 6.94 million

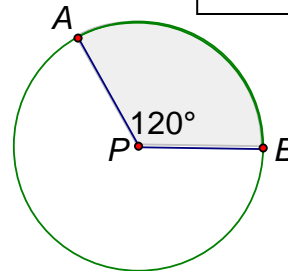
$$\frac{\text{degrees}}{\text{money}} = \frac{\text{degrees}}{\text{money}}$$

$$\frac{100}{x} = \frac{360}{25}$$

$$360x = 2500$$

$$x = 6.94$$

- 7) The area of the shaded sector shown is 75π . What is the circumference of the circle?



e) 30π

Area of a sector = $\frac{n}{360} \cdot \pi r^2$

$75\pi = \frac{120}{360} \cdot \pi r^2$ divide both sides by pi & reduce fraction

$75 = \frac{1}{3} r^2$ multiply both sides by 3

$225 = r^2$

$r = \sqrt{225}$

$r = 15$

Circumference = $d\pi$

If the radius is 15, then the diameter is 30.

$C = d\pi$

$C = 30\pi$

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E) AREA & VOLUME

FORMULAS FOR AREA

RECTANGLE



The area of a rectangle = length • width

SQUARE



The area of a square = (length • width) or (side)²

TRIANGLE



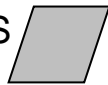
The area of a triangle = $\frac{1}{2}$ base • height

PARALLELOGRAM



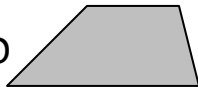
The area of a parallelogram = base • height

RHOMBUS



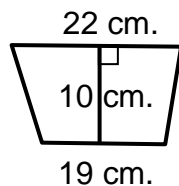
The area of a rhombus = base • height
or $\frac{1}{2}$ (diagonal₁ • diagonal₂)

TRAPEZOID



The area of a trapezoid = $\frac{1}{2}$ (base₁ + base₂) height

1) What is the area of the trapezoid shown below?



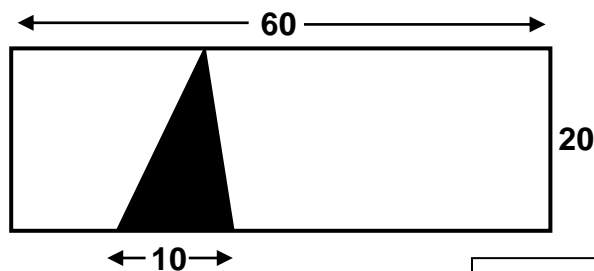
c) 205

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(22 + 19)(10)$$

$$A = 205$$

2) What is the area of the shaded triangle in the rectangle below?



100

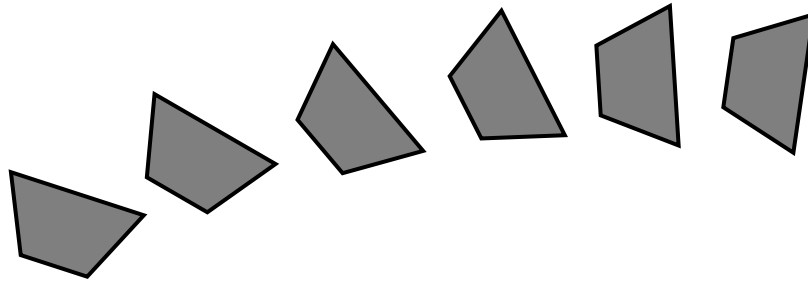
The height of the triangle is the same as the height of the rectangle = 20.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(20)$$

$$A = 100$$

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- 3) A garden path is made up of six congruent trapezoidal stepping stones as shown above. Each stone has bases of 8 in. and 10 in. and heights of 6 in. Find the total area of the stones.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(8 + 10)(6)$$

$$A = 54 = \text{area of 1 stone}$$

$$54(6) = 324 = \text{area of 6 stones}$$

324



- 4) Buddy is getting ready to roll his lawn with the lawn roller shown above. The diameter of the roller is a half of a foot. Approximately how many square feet will the roller cover in two rotations?

b) 9.42

1 rotation = 1 circumference x 3 ft.

$$C = d\pi$$

$$\text{diameter} = \frac{1}{2} \text{ foot} = .5$$

$$C = (.5)(3.14) = 1.57$$

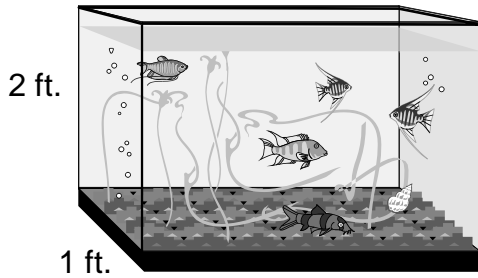
(1.57)(3) = 4.71 = area covered in one rotation. But, they asked for two rotations so we still have to multiply by 2!!

$$4.71(2) = 9.42$$

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VOLUME OF A RECTANGULAR SOLID (aka box)
 $V = L \cdot W \cdot H$

- 1) An aquarium tank has a volume of 10 cubic feet. If it is 1 foot wide, and 2 feet high, what is its length?



b) 5

$$V = L \cdot W \cdot H$$

$$10 = (L)(2)(1)$$

$$10 = 2L$$

$$L = 5$$

- 2) A driveway is 40 feet by 6 feet by 1/18 feet. How much concrete would be needed to fill the driveway?

d) 13.3 cu. ft

$$V = L \cdot W \cdot H$$

$$V = (40)(6)\left(\frac{1}{18}\right)$$

$$V = \frac{240}{18}$$

$$V = 13.3$$

- 3) The formula for volume of a pyramid is:

$V = \frac{1}{3}Bh$ where B represents the area of the base and h represents the height.

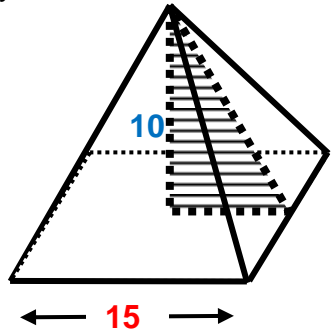
Find the volume of this **square** pyramid:

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(s^2)h$$

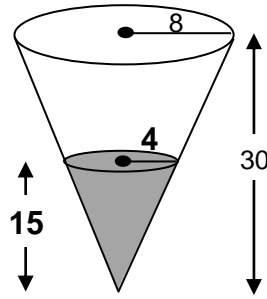
$$V = \frac{1}{3}(15^2)(10)$$

$$V = \frac{1}{3}(225)10 = 750$$



750

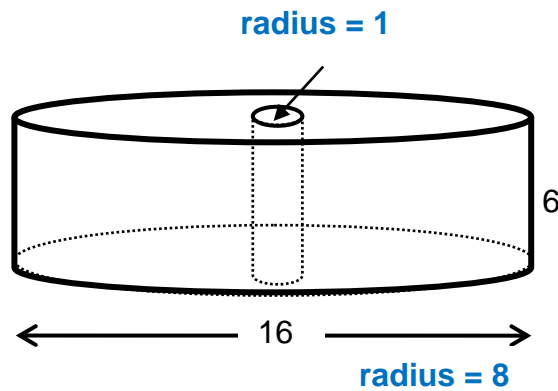
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- 4) The figure above shows a right circular cone with base radius 8 and height 30. The shaded portion of the figure is a right circular cone with height 15. The volume of the smaller cone is what **fraction** of the volume of the larger cone?
 (The volume of a right circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.)

$$\frac{V_{\text{small}}}{V_{\text{large}}} = \frac{\frac{1}{3}\pi r_s^2 h_s}{\frac{1}{3}\pi r_L^2 h_L} = \frac{\frac{1}{3}\pi(4)^2(15)}{\frac{1}{3}\pi(8)^2(30)} = \frac{16(15)}{64(30)} = \frac{1}{8}$$

$$\frac{1}{8}$$

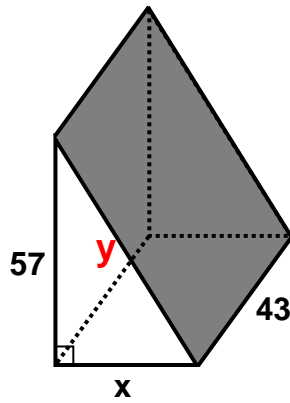


- 5) The cylinder above has a hole with a **2 inch diameter** cut out of the center. What is the volume of the remaining solid? $V = \pi r^2 h$

a) **378π**

$$\begin{aligned} V_{\text{Remaining}} &= V_{\text{Large}} - V_{\text{Small}} \\ V_R &= \pi r_L^2 h - \pi r_s^2 h \\ V_R &= \pi(8)^2(6) - \pi(1)^2(6) \\ V_R &= 384\pi - 6\pi \\ V_R &= 378\pi \end{aligned}$$

LESSON 3 **KEY** – GEOMETRY



- 6) A right triangular prism is shown above. If the area of the shaded rectangle is 2924, what is the value of x to the nearest tenth?

First find the length of the rectangle. I'll call it y .

Area of a rectangle = length \times width

Let L = the length

$$2924 = 43L \text{ divide by } 43$$

$$L = 68$$

Then use the Pythagorean Theorem to find x .

$$a^2 + b^2 = c^2$$

$$x^2 + 57^2 = 68^2$$

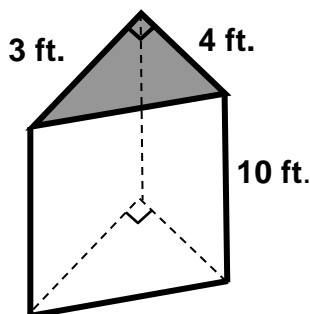
$$x^2 + 3249 = 4624$$

$$x^2 = 1375$$

$$x = \sqrt{1375} = 37.08 = \mathbf{37.1} \text{ to the nearest 10th}$$

37.1

- 7) Find the volume of the triangular prism below. The volume of a prism is $V = Bh$ where B represents the area of the base.



$$V = Bh$$

Since the base is a triangle,

B = formula for area of a triangle.

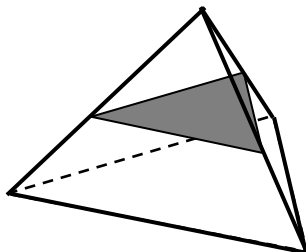
$$V = \left(\frac{1}{2} b_{\triangle} h_{\triangle} \right) h_{\text{prism}}$$

$$V = \left(\frac{1}{2} \cdot 3 \cdot 4 \right) 10$$

$$V = \mathbf{60}$$

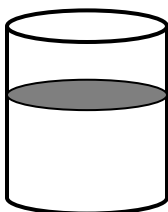
60

F) CROSS-SECTIONS



- 1) Which figure would represent a cross-section that is parallel to the base of the pyramid above?

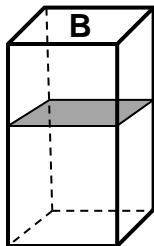
d) triangle



- 2) Which figure would represent a cross-section that is parallel to the base of the cylinder above?

b) circle

- 3) Candice cuts out a “slice” of the solid with a plane parallel to the base (B), covers it with ink and then presses it to a piece of paper. Which of the following shows the shape of the ink on the paper?



b)

